

Wither the sliding Luttinger liquid phase in the planar pyrochlore

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Using series expansion based on the flow equation method we study the zero temperature properties of the spin-1/2 planar pyrochlore antiferromagnet in the limit of strong diagonal coupling. Starting from the limit of decoupled crossed dimers we analyze the evolution of the ground state energy and the elementary triplet excitations in terms of two coupling constants describing the inter dimer exchange. In the limit of weakly coupled spin-1/2 chains we find that the fully frustrated inter chain coupling is critical, forcing a dimer phase which adiabatically connects to the state of isolated dimers. This result is consistent with findings by O. Starykh, A. Furusaki and L. Balents (Phys. Rev. B **72**, 094416 (2005)) which is inconsistent with a two-dimensional sliding Luttinger liquid phase at finite inter chain coupling.

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INTRODUCTION

Geometrically frustrated quantum magnets above one dimension (1D) have attracted strong interest in recent years [1]. From an experimental point of view this is due in part to the discovery of materials like the two-dimensional (2D) spin-dimer system $\text{SrCu}_2(\text{BO}_3)_2$ [2] and the three-dimensional (3D) tetrahedral compounds $\text{Cu}_2\text{Te}_2\text{O}_5\text{X}_2$ with $\text{X}=\text{Cl}, \text{Br}$ [3, 4] which exhibit strong frustration of the magnetic exchange. From a theoretical point of view, the prime interest in such systems stems from the possibility of realizing quantum disordered states and spin liquid behavior. Particularly promising candidates in this direction are systems which comprise tetrahedra coupled into 2D or 3D networks. Indeed, experimental observations on pyrochlore materials, like CsNiCrF_6 [5, 6] or $\text{Tb}_2\text{Ti}_2\text{O}_7$ [7] suggest the possibility of liquid-like behavior. In contrast to the latter systems, which are essentially classical due to Ising anisotropy and spins larger than 1/2, the $SU(2)$ invariant spin-1/2 Heisenberg antiferromagnet on the 3D pyrochlore lattice is an open issue. Experiments on the $S = 1/2$ system $\text{Y}_2\text{Ir}_2\text{O}_7$ [8] seem to indicate the presence of quantum spin liquid state. To reduce the complexity of the 3D system, a 2D projection, i.e. the planar pyrochlore, also known as checkerboard or crossed chain model (CCM), which is shown in Fig.1 for $J_p = J_d = J_0$, has been considered as a simplified starting point. The CCM model and some of its anisotropic generalizations have been studied by employing a variety of techniques. Analytic methods include semiclassical $S \gg 1$ analysis [9, 10, 11], large-N limit of the $\text{Sp}(N)$ model [12, 13, 14], series expansion (SE) by unitary transformation [15, 16] and quantum field theory (QFT) methods [17, 18]. Besides, this model has been studied extensively by means of exact diagonalization (ED) techniques [19, 20, 21]. These investigations have given a fairly complete picture of the phases of the system at different sectors of parameters space. At the checkerboard point ($J_p = J_d = J_0$) a

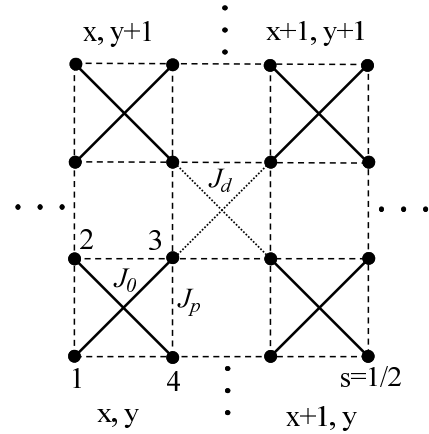


FIG. 1: The *dimerized* planar pyrochlore lattice. Spin-1/2 moments are located on solid circles. Crossed dimers at site x, y (bold solid lines) are locally coupled via J_p (dashed lines) and non-locally coupled to nearest neighbors dimer by J_p and J_d (dotted lines). On dimer exchange J_0 will be set to unity hereafter.

valence bond crystal (VBC) ground state with a spin gap has been found [15, 16, 20, 21]. For $J_d = J_0 \rightarrow 0$ model of Fig.1 tends to the 2D spin-1/2 Heisenberg model on the square lattice, which is known to have gapless magnon excitations and AFM-LRO. From this limit, by means of a phenomenological approach within the framework of the Landau-Ginzburg-Wilson (LGW) theory, it has been suggested recently that the transition between the (AFM-LRO) phase and the VBC, as $J_d (= J_0)$ are increased, could happen within a coexistence region or by a first order transition [18] but this point deserves more investigation. Previous results from ED up to 36 spins for $J_d = J_0$ in ref.[20] predict a finite gap at $J_d/J_p = 0.85$ and an extrapolated closure of the gap at $J_d/J_p = 0.65$. Furthermore, semiclassical approaches predict stability of the AFM-LRO phase, in the case of $S = 1/2$ at $J_d = J_0$ for $J_d/J_p \leq 0.76$.

In the opposite limit of weakly coupled spin-1/2 chains, i.e. $J_p \ll J_d = J_0$ in Fig.1, there are contro-

versal results. On the one hand, previous findings based on QFT methods [17] suggested that the model exhibits a 2D spin liquid ground state, being an example of the so called *sliding Luttinger liquid* (SLL) which is characterized by the absence of LRO and the elementary excitations being massless, deconfined spinons. It has been suggested that the prime reason for this behavior stems from the momentum dependence of the completely frustrated inter chain coupling. The latter vanishes exactly at the AFM nesting vector, suppressing a conventional magnetic instability. These results were corroborated subsequently by numerical studies [20], where a range $J_p/J_d \in [0, \simeq 0.8]$ for the extension of an effective 1D behavior was suggested.

On the other hand, by employing bosonization and renormalization group methods, Starykh et al. [18] have recently suggested that instead of exhibiting a SLL phase, the model of Fig.1 enters a gapped *crossed dimer* phase for $J_p \ll J_d = J_0$, with a spin gap Δ which opens according to $\Delta/J_d \simeq 0.675J_p^2$.

In this situation it is desirable to obtain results from complementary methods, which will be the primary goal of this work. Therefore, in the following analysis we will employ a SE in terms of J_p and J_d as in Fig.1 and will evaluate the ground state energy and the elementary excitations. Our findings will support the existence of a crossed dimer phase, in agreement with the results of ref.[18].

To perform a SE, applicable in the vicinity of the anticipated SLL phase we decompose the planar pyrochlore lattice into a set of crossed, *dimerized* chains as shown in Fig.1. I.e. the model represents a 2D array of AFM coupled crossed spin-1/2 dimers. We will set $J_0 = 1$ hereafter. The Hamiltonian reads

$$H = H_0 + V; \quad H_0 = \sum_{x,y} h_0; \quad V = \sum_{x,y} (V_p + V_d), \quad (1)$$

in which h_0 refers to the local crossed dimer unit at site x, y , given by

$$\begin{aligned} h_0 &= (\mathbf{S}_1 \cdot \mathbf{S}_3 + \mathbf{S}_2 \cdot \mathbf{S}_4) \\ &\quad + J_p (\mathbf{S}_1 \cdot \mathbf{S}_2 + \mathbf{S}_2 \cdot \mathbf{S}_3 + \mathbf{S}_3 \cdot \mathbf{S}_4 + \mathbf{S}_4 \cdot \mathbf{S}_1) \\ &= \frac{1}{2} (\mathbf{S}_{13}^2 + \mathbf{S}_{24}^2 - 3) + \frac{J_p}{2} (\mathbf{S}_{1234}^2 - \mathbf{S}_{13}^2 - \mathbf{S}_{24}^2), \end{aligned} \quad (2)$$

where all spins are at site x, y and $\mathbf{S}_{1\dots n} = \mathbf{S}_1 + \dots + \mathbf{S}_n$. V_p and V_d in Eq.(1) represent the couplings between the crossed dimers at site x, y with their next nearest neighbors via J_p and J_d respectively, i.e.

$$\begin{aligned} V_p &= J_p [\mathbf{S}_{4;x,y} \cdot \mathbf{S}_{1;x+1,y} + \mathbf{S}_{3;x,y} \cdot \mathbf{S}_{2;x+1,y} \\ &\quad + \mathbf{S}_{2;x,y} \cdot \mathbf{S}_{1;x,y+1} + \mathbf{S}_{3;x,y} \cdot \mathbf{S}_{4;x,y+1}]; \\ V_d &= J_d [\mathbf{S}_{3;x,y} \cdot \mathbf{S}_{1;x+1,y+1} + \mathbf{S}_{2;x,y} \cdot \mathbf{S}_{4;x-1,y+1}]. \end{aligned} \quad (3)$$

Table I depicts the eigenstates of h_0 . From Eq.(2) it is

	S_{1234}	S_{13}	S_{24}	$q = E + 3/2$
$ \mathcal{S}_0\rangle$	0	0	0	0
$ \mathcal{T}_p\rangle$	1	1	0	1
$ \mathcal{T}_n\rangle$	1	0	1	1
$ \mathcal{S}_t\rangle$	0	1	1	$2 - 2J_p$
$ \mathcal{T}_t\rangle$	1	1	1	$2 - J_p$
$ \mathcal{Q}_t\rangle$	2	1	1	$2 + J_p$

TABLE I: Eigenstates of a local crossed dimer unit h_0 at site x, y . Each state is labelled by the quantum numbers: S_{1234} , S_{13} and S_{24} and the energy E .

clear that each state of h_0 can be labelled by the following quantum numbers: total spin S_{1234} , the spin of the dimer on the positive(negative) diagonal, i.e. the $S_{13}(S_{24})$ and the energy E . As one can observe, for $0 \leq J_p < 1$ the ground state is $|\mathcal{S}_0\rangle$, i.e. both dimers are singlets. Besides, for $0 \leq J_p < 1/2$ the first excited states are $|\mathcal{T}_p\rangle(|\mathcal{T}_n\rangle)$ i.e. one of the dimers on the positive(negative) diagonal is a triplet while the one on the other diagonal is a singlet. The other states are: a total spin singlet $|\mathcal{S}_t\rangle$, a triplet $|\mathcal{T}_t\rangle$ and a quintet $|\mathcal{Q}_t\rangle$, all of them consisting of triplets on both diagonals. Only the energies of the latter three states depend on J_p .

SERIES EXPANSION BY CONTINUOUS UNITARY TRANSFORMATION

In this section we briefly describe the SE expansion in terms of V . To this end we rewrite Eq.(1) as

$$H = H_0(J_p = 0) + J_p O_{p_0}^l + \sum_{n=-N}^N (J_p O_{p_n} + J_d O_{d_n}). \quad (4)$$

$H_0(J_p = 0)$ has an equidistant ladder spectrum (Table I) which we label with the *total particle-number* operator: $Q = \sum_{x,y} q(J_p = 0)$. The *vacuum* corresponds to the $Q = 0$ sector: $|0\rangle \equiv \prod_{x,y} |\mathcal{S}_{0;x,y}\rangle$. *One-particle* states correspond to the $Q = 1$ sector, whose basis states $|T_{\mu;x,y}\rangle (\mu = p, n)$ are composed of a local triplet on the positive or negative diagonal excited from the vacuum: $|T_{\mu;x',y'}\rangle \equiv |\mathcal{T}_{\mu;x',y'}\rangle \otimes \prod_{x,y \neq x',y'} |\mathcal{S}_0;x,y\rangle$. The $Q \geq 2$ sector is of multi-particle nature.

The $O_{p_0}^l$ operators in Eq.(4) refer to the local contributions proportional to J_p in H_0 . Finally, the third term in the same Eq. represents the inter dimer coupling via V . The O_{p_n} and O_{d_n} operators [22] non-locally create(destroy) $n > 0 (< 0)$ energy quanta within the ladder spectrum of $H_0(J_p = 0)$. Note that only terms containing two dimers at different x, y sites are present in V . This imposes the constraint $N \leq 2$.

Models of type Eq.(4) allow for SE using the flow equation method of Wegner [23, 24]. There, by means of a continuous unitary transformation, one maps $H \rightarrow H_{eff}$

with

$$H_{eff} = H_0 + \sum_{r, 0 \leq s \leq r} C_{r,s} J_p^{r-s} J_d^s, \quad (5)$$

in which the $C_{r,s}$ operators involve $r - s$ and s products of type- O_{p_n} and O_{d_n} operators, respectively. We refer to [24] for further details.

The main advantage of the flow equation method is that unlike H , the effective Hamiltonian H_{eff} is constructed to *conserve* the total number of particles Q of $H_0(J_p = 0)$ at each order in the expansion. This allows for the SE of several quantities from the bare eigenstates of $H_0(J_p = 0)$ as we show in the following sections.

GROUND STATE ENERGY

The ground state energy allows for an important consistency check of our calculation with respect to existing literature and other SE results. As mentioned above, there is evidence for a *quadramer* VBC phase of the CCM at the checkerboard point ($J_p = J_d = 1$). This is based on ED [21] and SE starting from the quadramer limit of the CCM [15, 16]. Moreover, Starykh et al. [18], have predicted a first order transition or a coexistence phase at some intermediate value of $0 < J_p < 1$ and $J_d = 1$ between the VBC and a crossed dimer phase. Therefore, in order for the present SE to give results consistent with them for J_p and J_d in the vicinity of the checkerboard point, we expect the ground state energy of the crossed dimer state to be larger than that of the quadramer VBC at $J_p = J_d = 1$.

For $0 \leq J_p < 1$ the ground state of $H_0(J_p = 0)$ is the $Q = 0$ sector, i.e. $|0\rangle \equiv \prod_{x,y} |\mathcal{S}_0\rangle_{x,y}$. Q -conservation of H_{eff} allows for the calculation of the evolution of this state as a $SE(J_p, J_d)$ by means of

$$E_g = \langle 0 | H_{eff} | 0 \rangle. \quad (6)$$

The actual calculation can only be done on finite clusters. To obtain results to $O(J_p^n, J_d^n)$, valid in the thermodynamic limit, i.e. for infinite sized systems, matrix elements given by Eq.(6) have to be evaluated on clusters with periodic boundary conditions (PBC), sufficiently large to avoid a wrap-around at graph length n [25]. In our case, the size of the Hilbert space, considering both perturbations J_p and J_d , restricts the calculation at $n = 5$ due to memory limitations. We find this enough to quantitatively describe the combined effects of J_p and J_d in the range of interest. At $O(J_p^5, J_d^5)$ the ground state energy per spin and PBC, $e_g(J_p, J_d)$, is given by

$$e_g(J_p, J_d) = -\frac{3}{8} - \frac{3}{64}J_d^2 - \frac{3}{32}J_p^2 - \frac{3}{256}J_d^3 + \frac{9}{128}J_dJ_p^2 - \frac{3}{128}J_p^3 - \frac{13}{4096}J_d^4 - \frac{7}{1024}J_d^2J_p^2 + \frac{5}{128}J_dJ_p^3 - \frac{47}{2048}J_p^4$$

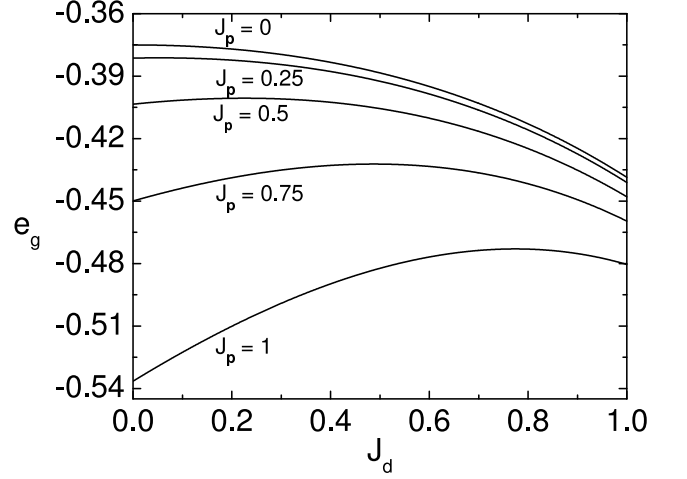


FIG. 2: Ground state energy $e_g(J_p, J_d)$ per spin vs J_d for different values of J_p , Eq.(7). As observed $e_g(J_p, J_d)$ monotonously decreases as J_p is increased from 0 to 1, for fixed values of J_d . Eq.(7) at the point of decoupled uniform spin-1/2 chains, i.e. $e_g(J_p = 0, J_d = 1)$, differs $\approx 1\%$ from the exact value $1/4 - \ln 2$ from Bethe ansatz. At the checkerboard point Eq.(7) gives $e_g(J_p = 1, J_d = 1) \approx -0.48$ which is above the value obtained by employing a SE starting from a different point in parameters space (see text). This is consistent with the stability of the quadramer VBC in the vicinity of the checkerboard point which has $\bar{e}_g \approx -0.51$.

$$-\frac{89}{49152}J_d^5 - \frac{307}{24576}J_d^3J_p^2 - \frac{71}{12288}J_d^2J_p^3 + \frac{869}{24576}J_dJ_p^4 - \frac{175}{8192}J_p^5. \quad (7)$$

As a check, note that $e_g(J_p = 0, J_d)$ reduces to the $O(J_d^5)$ dimer expansion of the ground state energy of the dimerized spin-1/2 chain [24, 26].

From Bethe ansatz [27] the ground state energy for the homogeneous spin-1/2 chain is known: $\tilde{e}_g(J_p = 0, J_d = 1) = 1/4 - \ln 2$ [28]. Comparing it with $e_g(J_p = 0, J_d = 1)$ from Eq.(7) we obtain a relative error of $\approx 1\%$, which gives a rough estimation of the convergence [29].

Fig.2 depicts the evolution of the ground state energy as given by Eq.(7) vs J_d for different values of J_p . As shown, $e_g(J_p, J_d)$ monotonously decreases as J_p is incremented from 0 to 1, for fixed values of J_d . The position J_{d_m} of the maximum of $e_g(J_p, J_d)$ vs J_d shifts from $J_{d_m} = 0$ to $J_{d_m} \approx 0.8$ for J_p taking fixed values from 0 to 1, respectively. At the checkerboard point our SE yields $e_g(J_p = 1, J_d = 1) \approx -0.48$. Note that our value is larger than the value of $\bar{e}_g \approx -0.51$ which has been obtained from the quadramer SE in ref.[15]. While the expansion parameters and their respective orders cannot be compared directly between the present SE and that of ref.[15] these findings are at least consistent with the stability of the quadramer VBC in the vicinity of the checkerboard point. This, together with the fact that $H_0(J_p = 1)$ has a degenerated ground state (see Table

1), suggests that Eq.(7), at least at the checkerboard point, does not represent the ground state energy of the system anymore, which is $\bar{\epsilon}_g \approx -0.51$.

TRIPLET DISPERSION

Next we evaluate the dispersion of the one-triplet states. To this end it is necessary to diagonalize H_{eff} in the $Q = 1$ sector of $H_0(J_p)$, i.e. the subspace spanned by the vectors $|T_{\mu;x,y}\rangle, \mu = \{p, n\}$. First we switch to momentum space

$$|T_{\mu;k_x,k_y}\rangle = \sqrt{\frac{1}{L^2}} \sum_{x,y} [\exp(ik_x x + iyk_y) |T_{\mu;x,y}\rangle], \quad (8)$$

in which PBC is assumed and L^2 is the number of sites. Because of the degeneracy of the $Q=1$ sector, H_{eff} is a 2×2 matrix in the basis given by Eq.(8), whose entries $E_{T\mu,\nu;k_x,k_y}$ are

$$\begin{aligned} E_{T\mu,\nu;k_x,k_y} &= \langle T_{\mu;k_x,k_y} | H_{eff} | T_{\nu;k_x,k_y} \rangle \\ &= \sum_{x,y} [c_{\mu,\nu;x,y} \exp(ik_x x + iyk_y)], \end{aligned} \quad (9)$$

where $\mu, \nu = \{p, n\}$ and

$$c_{\mu,\nu;x,y} = \langle T_{\mu;x,y} | H_{eff} | T_{\nu;0,0} \rangle \quad (10)$$

are the hopping amplitudes from a ν -type of triplet on site $0,0$ to a μ -type of triplet on site x,y . Evaluation of the $c_{\mu,\nu;x,y}$'s is facilitated by symmetry which leaves only a subset of them to be independent. To obtain the hopping amplitudes valid in the thermodynamic limit, the $c_{\mu,\nu;x,y}$'s are calculated on finite clusters with open boundary conditions (OBC), and a cluster-topology suitable to embed all the paths of length n connecting the site $(0,0)$ with (x,y) at $O(n)$ of the perturbation. From this, the triplet dispersion is obtained as

$$\omega(\mathbf{k}, J_p, J_d) = E_T(\mathbf{k}, J_p, J_d) - \langle 0 | H_{eff} | 0 \rangle, \quad (11)$$

where $\mathbf{k} = (k_x, k_y)$ and the second term refers to the ground state energy which has to be evaluated on the identical cluster as $c_{\mu,\nu;0,0}$. The triplet energy $E_T(\mathbf{k}, J_p, J_d)$ results from diagonalizing the 2×2 matrix of Eq.(9), leading to two excitation branches.

We have calculated the triplet dispersion $\omega(\mathbf{k}, J_p, J_d)$ up to $O(J_p^5, J_d^5)$ [30]. Upon rotating $k = (k_x, k_y) \rightarrow (k_p, k_n)$ along the positive and negative diagonals, by means of $k_{x,y} = (k_p \mp k_n)/2$, the two branches of Eq.(11) have to reduce to the triplet dispersion of the dimerized spin-1/2 chains along positive and negative diagonals for $J_p = 0$. For consistency, we have checked that this is satisfied up to the order of our SE by comparing with existing SE results on the single dimerized chain [24, 26].

TRIPLET GAP

Using the results from the previous section we may now turn to one of the main objectives of this work, i.e. to study the evolution of the triplet gap under the combined effect of J_p and J_d . This will enable us to conclude on the stability of the dimerization along the chains and therefore to discriminate between the controversial proposals on the stability of the SLL. The triplet gap: $\Delta(J_p, J_d) = \min \omega(\mathbf{k}, J_p, J_d)$ is obtained from the minimum in \mathbf{k} -space of the two one-triplet branches. For $J_p \leq 0.5$ we find the triplet gap to be at $k_p = k_n = 0$. It is given by

$$\begin{aligned} \Delta(J_p, J_d) &= 1 - \frac{1}{2}J_d - \frac{3}{8}J_d^2 + \frac{1}{32}J_d^3 + \frac{29}{32}J_d J_p^2 - \frac{3}{32}J_p^3 \\ &\quad - \frac{5}{384}J_d^4 - \frac{401}{384}J_d^2 J_p^2 + \frac{17}{32}J_d J_p^3 - \frac{53}{128}J_p^4 - \frac{761}{12288}J_d^5 \\ &\quad + \frac{1309}{1536}J_d^3 J_p^2 - \frac{3895}{4608}J_d^2 J_p^3 + \frac{6253}{3072}J_d J_p^4 - \frac{2065}{4608}J_p^5. \end{aligned} \quad (12)$$

We have analyzed $\Delta(J_p, J_d)$ both by its plain SE and by using a biased-Padé approximation[24]. The latter consists of an extrapolation biased by the known analytical form of the gap-closure of the dimerized spin-1/2 chain as a function of J_d [31], i.e. $\Delta(J_p = 0, J_d \rightarrow 1) \propto (1 - J_d)^{2/3}$. Using this, J_d is replaced in favor of $J_d = 1 - x^{3/2}$ in Eq.(12). Then, standard Padé approximation[32] is performed around $x = 1$ and finally $x = (1 - J_d)^{2/3}$ is re-inserted, arriving at a transformed Padé approximation in terms of the original variable J_d .

Fig.3 depicts the evolution of $\Delta(J_p, J_d)$ as a function of J_d for different values of J_p . Solid lines show the results from the plain series, i.e. Eq.(12) and dashed lines exhibit the biased-Padé(2,2) approximants. As the inset of Fig.3 shows, the approximant describes the closure of the gap at $J_p = 0, J_d = 1$ very well. This justifies the use of this extrapolation technique and shows that our expansion can be employed to analyze the coupling of the spin-1/2 chains via J_p .

As a first result from Fig.3, both the plain series and the biased-Padé clearly show an *opening* of the gap at $J_d = 1$ for increasing values of J_p . As it is also obvious from these plots, this tendency persists for all $J_d \geq 0.2$. Second, Fig.3 shows, that the biased Padé approximant leads to observable differences from the plain SE only in the vicinity of the gap-closure.

Now, we consider the opening of the spin gap at $J_d = 1$ for $J_p \ll 1$. To this end, we first shift the zero of energy in order to suppress the finite gap of the SE at $J_p=0$ and $J_d=1$, i.e. we consider $\Delta(J_p, J_d = 1) - \Delta(J_p = 0, J_d = 1)$. By this we account for the inability of the SE to provide for a truly gapless spectrum in the limit of decoupled AFM Heisenberg chains. As it is obvious from Fig.3, this shift is very small for the biased Padé, i.e. ≈ 0.003 and ≈ 0.081 for the plain SE. The resulting spin gap versus J_p is shown in Fig.4 where the solid line represents

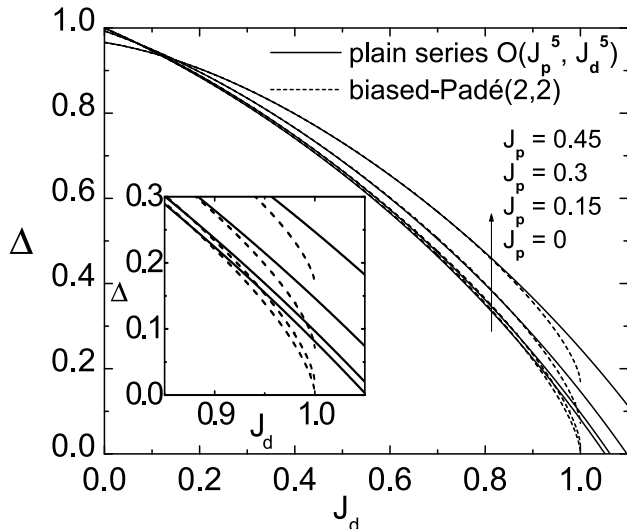


FIG. 3: Triplet gap Δ vs J_d for increasing values of J_p , Eq.(12) (solid lines) and biased-Padé(2,2) (dashed lines). As the inset shows, the biased-Padé(2,2) describes the closure of the gap at $(J_p = 0, J_d = 1)$ very well. The effect of J_p opening the gap is evident from the arrow.

the plain series and the dashed line corresponds to the biased-Padé(2,2) approximant.

These results may be compared with recent findings of Starykh et al. [18]. Using QFT methods, i.e. renormalization group and bosonization, they have obtained an analytic expression for the spin gap, which is quadratic in J_p , namely $\Delta(J_p \ll 1, J_d = 1) = cJ_p^2$ where $c \approx 0.675$. This function is shown in the same Fig.4 with a dash-dotted line. Obviously, there is a very good agreement up to $J_p \approx 0.1$. This is remarkable, given the completely different nature of our approach and that of ref.[18]. We note that the same agreement is found when we employ standard Dlog-Padé approximants instead of biased-Padé. Therefore, we are strongly tempted to claim that the stable phase of the CCM at finite J_p and at $J_d = 1$ is indeed gapped. This is inconsistent with a SLL state.

While Fig.4 seems to suggest an ever-increasing spin gap, one has to remember, that at $J_p = 0.5$ the nature of the first excited state of $h_0(J_p)$ changes from the triplet $|\mathcal{T}_{p,n}\rangle$ to the singlet $|\mathcal{S}_t\rangle$ (Table 1). This imposes an upper limit on the range of validity of the SE.

CONCLUSIONS

In summary, using series expansion, we have studied the zero temperature properties of the planar pyrochlore quantum magnet focussing on its behavior close to the crossed chain limit. In particular, we have considered the ground state energy, the elementary triplet dispersion and the spin gap.

In relation with the ground state energy calculations

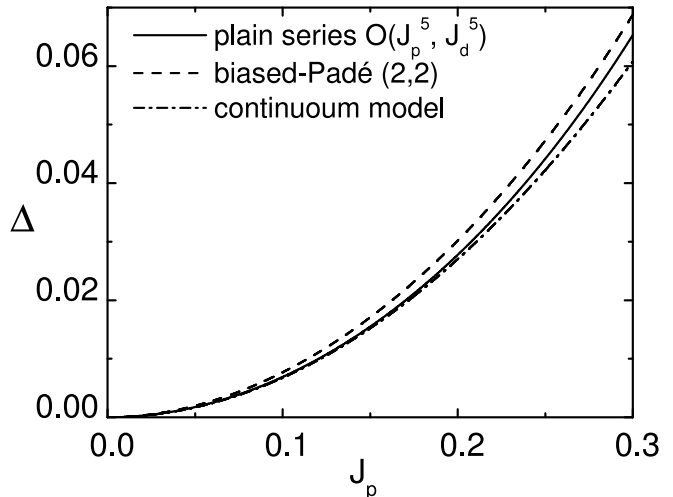


FIG. 4: Evolution of the triplet gap $\Delta(J_p, J_d = 1) - \Delta(J_p = 0, J_d = 1)$ vs J_p starting from the limit of decoupled spin-1/2 chains ($J_p = 0, J_d = 1$). Plain series, Eq.(12), (solid line) and biased-Padé(2,2) approximant (dashed line). The shift $\Delta(J_p = 0, J_d = 1)$ has no effect on the functional dependence on J_p and it is made for a better comparison with the QFT results by Starykh et al. [18], i.e. $\Delta(J_p \ll 1, J_d = 1) \approx 0.675J_p^2$, (dash-dotted line). Very good agreement between both techniques, up to $J_p \approx 0.1$, is observable. The same agreement is found by employing standard Dlog-Padé technique.

our findings are consistent with previous results regarding the stability of the quadrumer VBC at the checker-board point.

Our main result regards the existence of crossed dimer phase. Our calculations univocally show an opening of the gap $\Delta(J_p \ll 1, J_d = 1)$. Moreover, our expansion (Eq.(12)), allows to route the physical origin of the crossed dimer phase by showing that it is adiabatically connected with decoupled crossed dimer units.

On the other hand, the very good agreement with a recent field theory approach by Starykh et.al. [18] even for $J_p \leq 0.1$ consolidates the hypothesis about the existence of a crossed dimer phase in the region of parameters space $J_d = 1, J_p \ll 1$ and indicates the absence of Luttinger liquid behavior, the so called sliding Luttinger liquid phase, in this region of parameters space.

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